Warm-up!

These are review if you remember…

A) Let $f(x) = \frac{x}{3} - 1$. Find $f^{-1}(7)$.

B) Let $g(x) = x^3 - 4$. Find $g^{-1}(x)$.

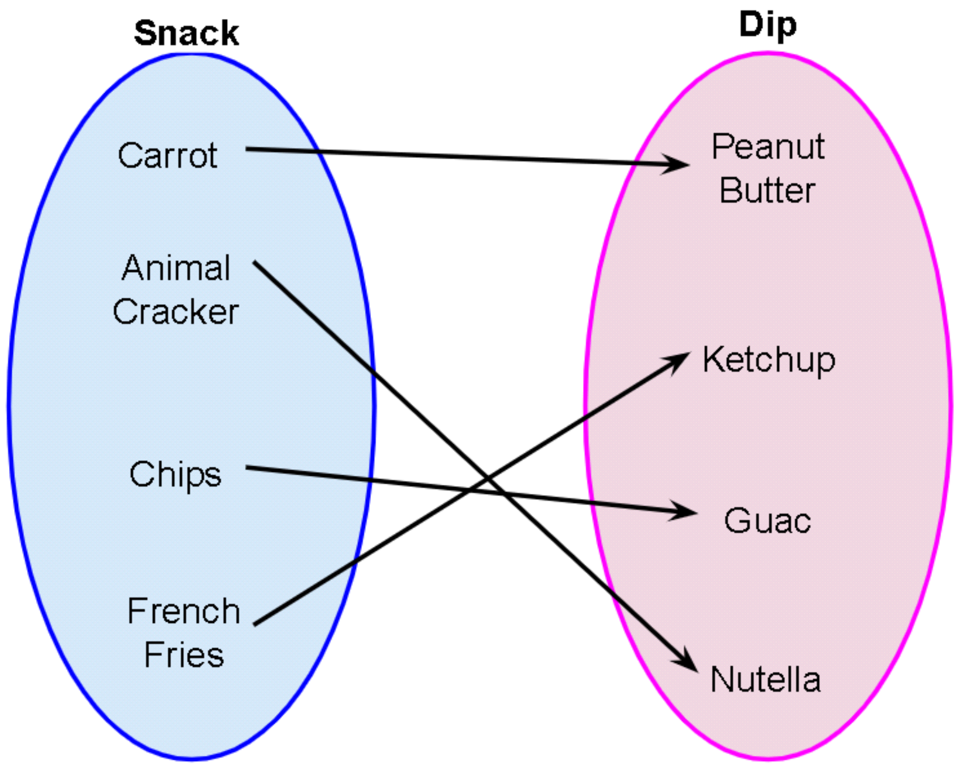
C) Let $h(x) = \frac{2x-1}{x+5}$. Find all asymptotes of $h^{-1}(x)$.

Periodic Graphs Day 10:
(2.8) Inverse Functions

Homework: Inverse Functions Practice

Upcoming:
Desmos Quiz on Day 12
Test on Day 13

One way to think about it: Inverse functions "undo" each other.



Consider the function $h(t)$ that inputs how many days it has been since the start of the school year and outputs how many million people have seen the movie K Pop Demon Hunter. For instance $h(5) = 203$ means that by five days after the school year began, 203 million people had already watched K Pop Demon Hunters.

What does $h^{-1}(250) = 20$ represent?

Consider the function f that inputs the number of people who stop at a gas station and outputs a prediciton for how many bags of chips the gas station will sell. And consider g that inputs the price of gas and outputs how many people will stop at the gas station.

What does f^{-1} input and output?

What does g^{-1} input and output?

What does $\left(g^{-1} \circ f^{-1}\right)(x)$ input and output?

Let $f(x) = 3x + 10$

Evaluate $f(4)$

Evaluate $f^{-1}(4)$

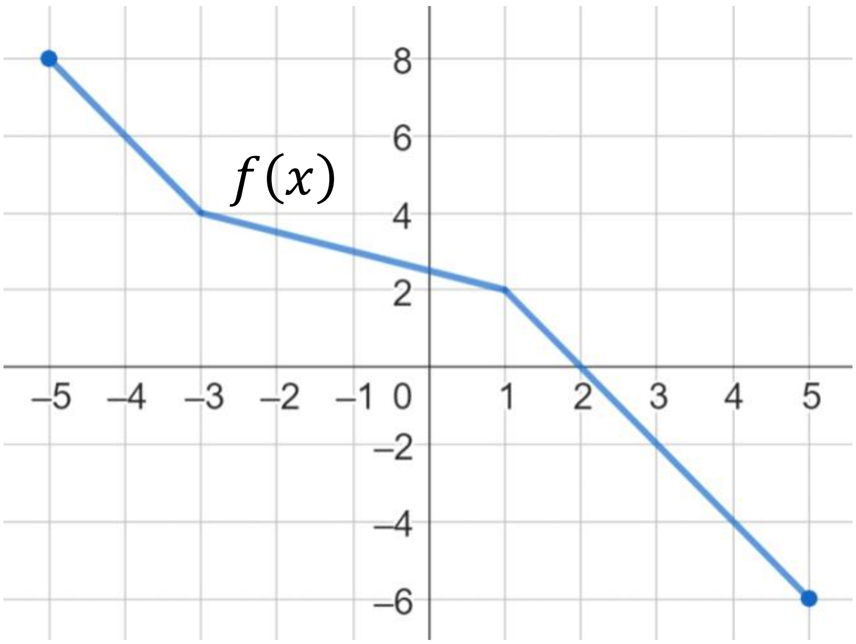
Let $g(x)$ be an invertible function (AKA "one-to-one") with values given in the table below.

x	$g(x)$
-2	1
0	2
1	3
3	5
5	7
6	10

A) Find $g^{-1}(3)$

B) Find $g^{-1}(1)$

C) Find $g^{-1}\left(g^{-1}(7)\right)$



A) $f^{-1}(2)$

B) $f^{-1}(4)$

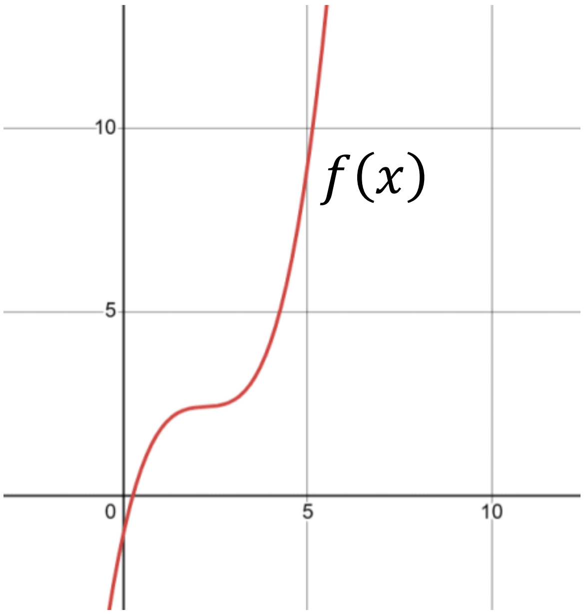
C) $\left(f^{-1} \circ g^{-1}\right)(2)$

D) $\left(g^{-1} \circ f\right)(3)$

Let $g(x) = \frac{12}{x}$ for $x \neq 0$

Let's try doing that with Desmos

Use Desmos to graph $f(x) = 0.3x^3 - 2x^2 + 4.5x - 1.$
The graph should look something like the graph shown below



A) $f^{-1}(6)$

B) $f^{-1}(1)$

C) $\left(f^{-1} \circ g^{-1}\right)(2)$

D) $\left(g^{-1} \circ f\right)(3)$

Let $g(x) = \sqrt{x+1}$ for $x > 0$

The x y switcheroo

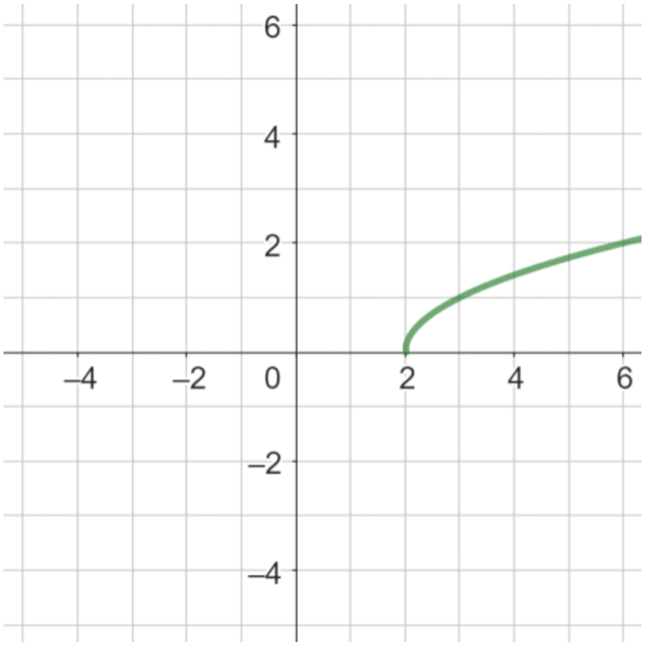
A) If the graph of $y = f(x)$ contains the point $(2,5)$, what do we know about the graph of $f^{-1}(x)$?

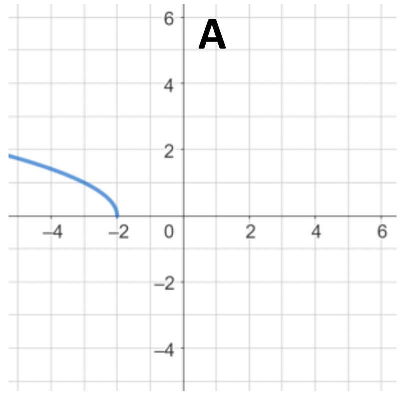
B) If the graph of $y = f(x)$ has domain $(-4, \infty)$, what do we know about the graph of $f^{-1}(x)$?

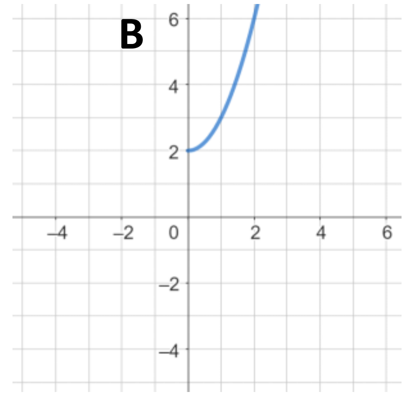
C) If the graph of $y = f(x)$ has range $[-10,4)$, what do we know about the graph of $f^{-1}(x)$?

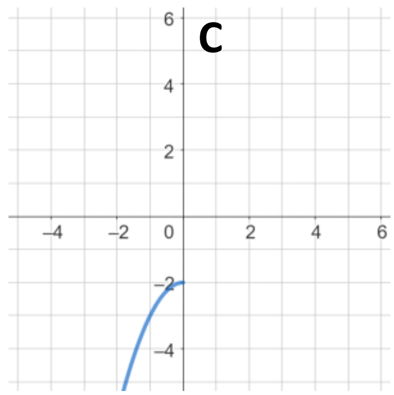
D) If the graph of $y = f(x)$ has asymptote $x = 4$, what do we know about the graph of $f^{-1}(x)$?

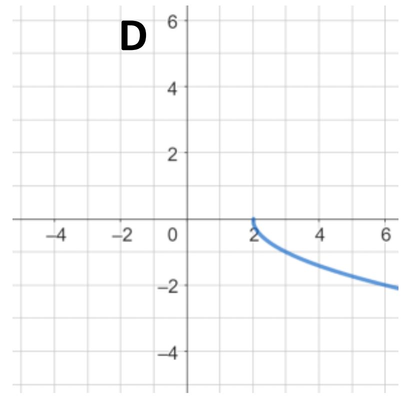
On the left is the graph of $f(x)$. Of the four smaller graphs to the right, determine which is the graph of $f^{-1}(x)$.

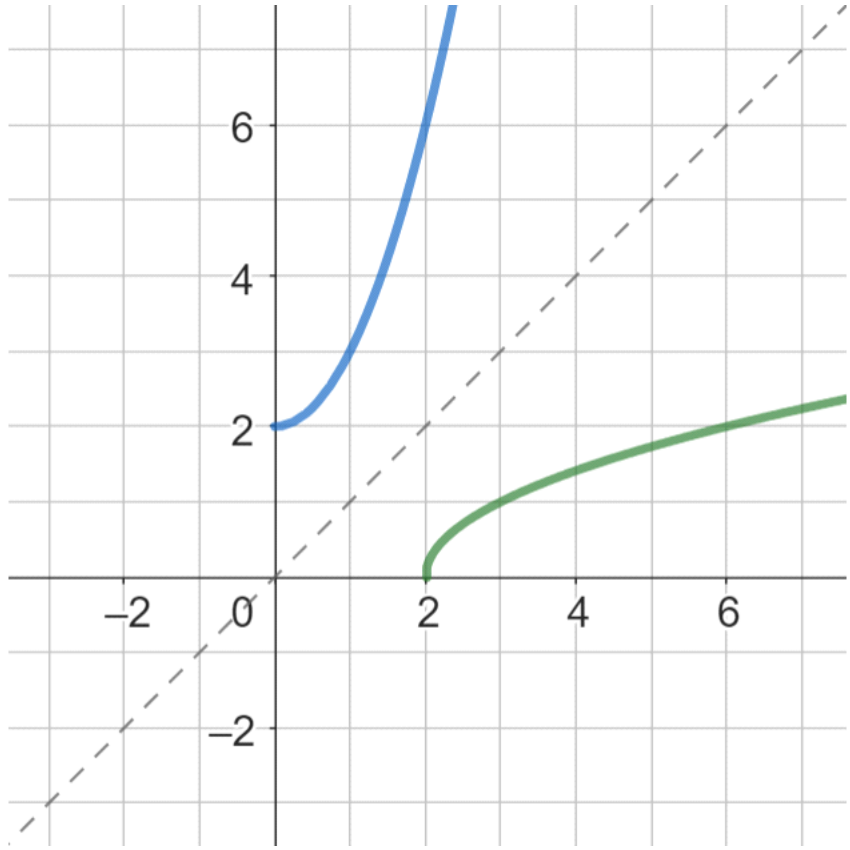






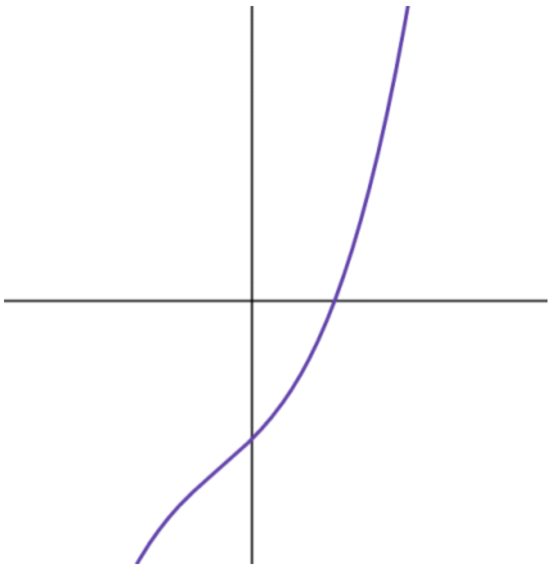





6
4
2
–2
–2
0
2
4
6

Below is the graph of $y = f(x) = 0.21x^3 + \frac{3}{10}x^2 + x - 2$

Use Desmos to obtain a graph of $f(x)$. It should look like the graph shown below.

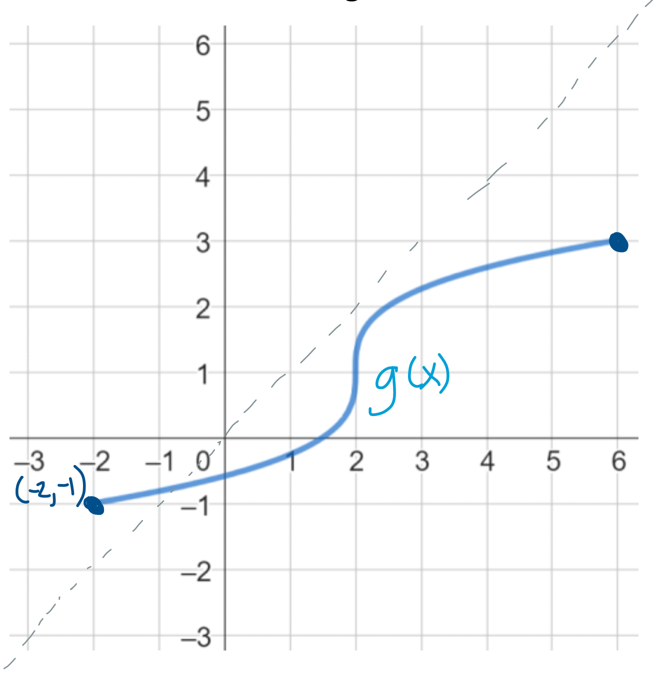


Roughly sketch what you believe the graph of $f^{-1}(x)$ would look like.

Using Desmos, find the y-intercept of $f^{-1}(x)$

Then, find the x-intercept of $f^{-1}(x)$

Below is the graph of $g(x)$ defined only on the interval $-2 \leq x \leq 6$



A) Roughly sketch g^{-1} on the graph of g

B) What is the absolute minimum value of g^{-1}?

C) Over what interval is g^{-1} increasing?

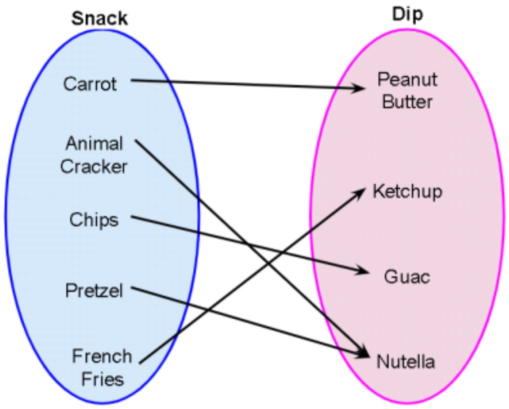
D) Over what interval is g^{-1} concave down?

So far we have learned/remembered

- When asked $f^{-1}(c)$, c is the output (y-value) of f and we need to find the corresponding input (x-value) of f.
- Comparing graphs of f and f^{-1}, the role of x and y swap
  - The point (a, b) becomes the point (b, a)
  - Domains become ranges, ranges become domains
  - Horizontal asymptotes $y = \#$ become vertical asymptotes $x = \#$
- Comparing graphs of f and f^{-1}, they look like reflections of each other over the line $y = x$.

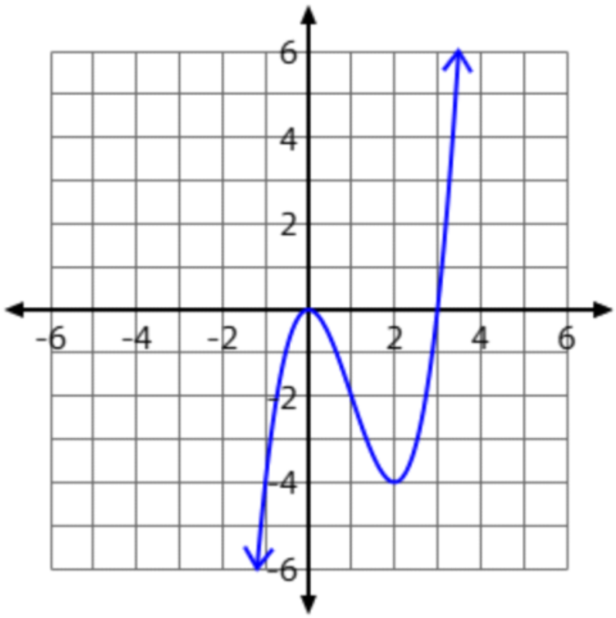
"One To One"

Not all functions have an inverse function



When a function does have an inverse function, we call it "one-to-one"

Consider the graph of $f(x)$ shown below

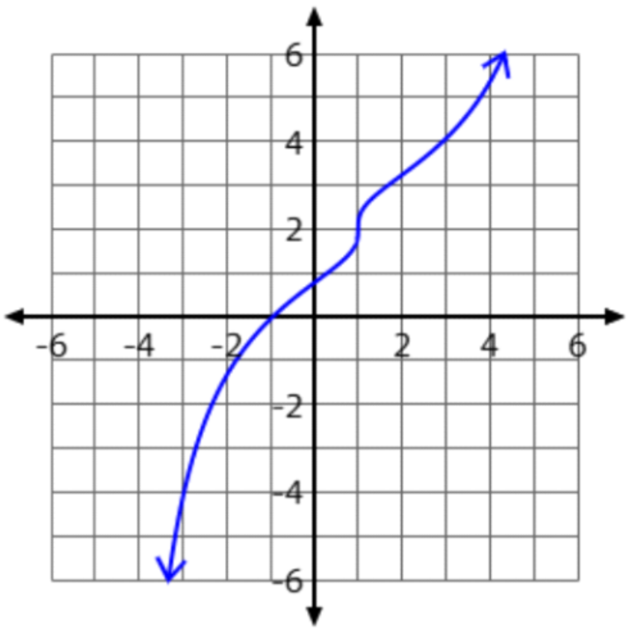


A) Determine if f has an inverse function

B) Give a reason based on the graph of f. Refer to specific values from the graph.

C) If f does not have an inverse, is there a way to restrict the domain so that f does have an inverse?

Consider the graph of $g(x)$ shown below

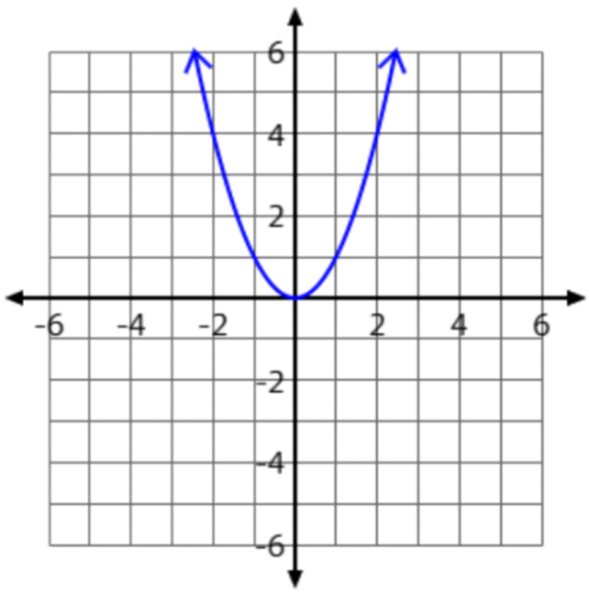


A) Determine if g has an inverse function

B) Give a reason based on the graph of g. Refer to specific values from the graph.

C) If g does not have an inverse, is there a way to restrict the domain so that g does have an inverse?

Consider the graph of $h(x)$ shown below



A) Determine if h has an inverse function

B) Give a reason based on the graph of h. Refer to specific values from the graph.

C) If h does not have an inverse, is there a way to restrict the domain so that h does have an inverse?

Proving Two Functions Are Inverses

You must show that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ (note: beyond this being a technique to show the functions are inverses, this is the definition of inverse functions)

Example:
Show $f(x) = 4x + 9$ and $g(x) = \frac{x-9}{4}$ are inverses.

Are $f(x) = \frac{3}{x} + 2$ and $g(x) = \frac{2}{x} - 3$ inverses? Prove it.

Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses? Prove it.

Finding Inverse Functions

Step 1: Swap x and y
Step 2: Isolate y
Step 3: Profit

Example: Let $f(x) = \frac{2x-1}{3}$. Find an expression for $f^{-1}(x)$.

Find the inverse, if possible

B) $g(x) = \sqrt[3]{x+8}$

Find the inverse, if possible

C) $t(x) = \frac{2}{x+1}$

Find the inverse, if possible

D) $h(x) = x^2 - 4x + 1$

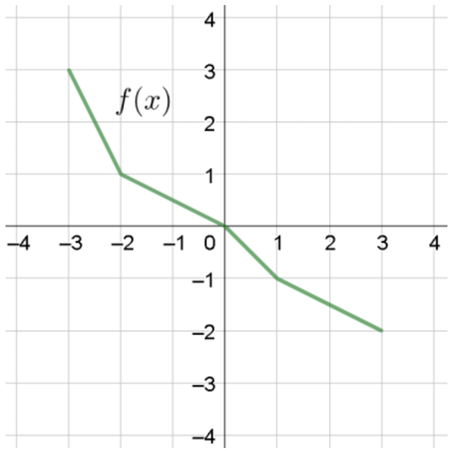
Find the inverse, if possible

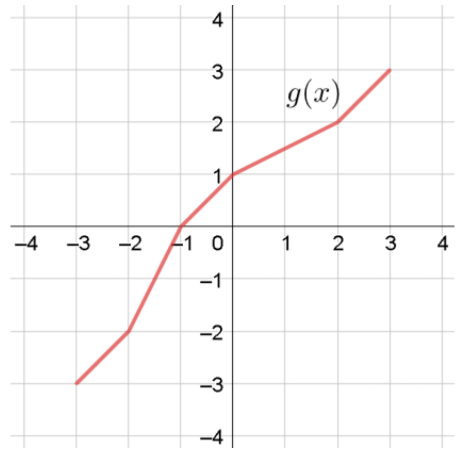
E) $k(x) = \frac{x}{x-4}$

More that we've learned/remembered today

- A function has an inverse if it passes both the vertical line test (VLT) and horizontal line test (HLT). The term for such a function is "one-to-one."
- To verify two functions are inverses of each other, show $(f \circ g)(x) = x$ AND $(g \circ f)(x) = x$
- Sometimes we have to restrict the domain of a function to make it invertible (AKA "to make it one-to-one")
- To find the inverse function equation, swap x and y, then solve for y.

Find the indicated values if possible





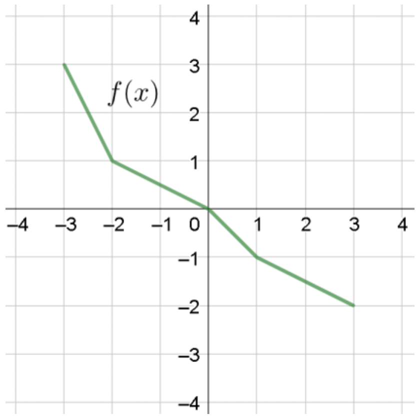
A) $f^{-1}(3)$

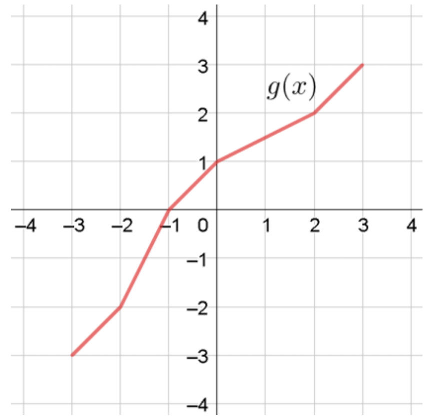
B) $g^{-1}(0)$

C) $g^{-1}(1)$

D) $f^{-1}(0)$

Find the indicated values if possible



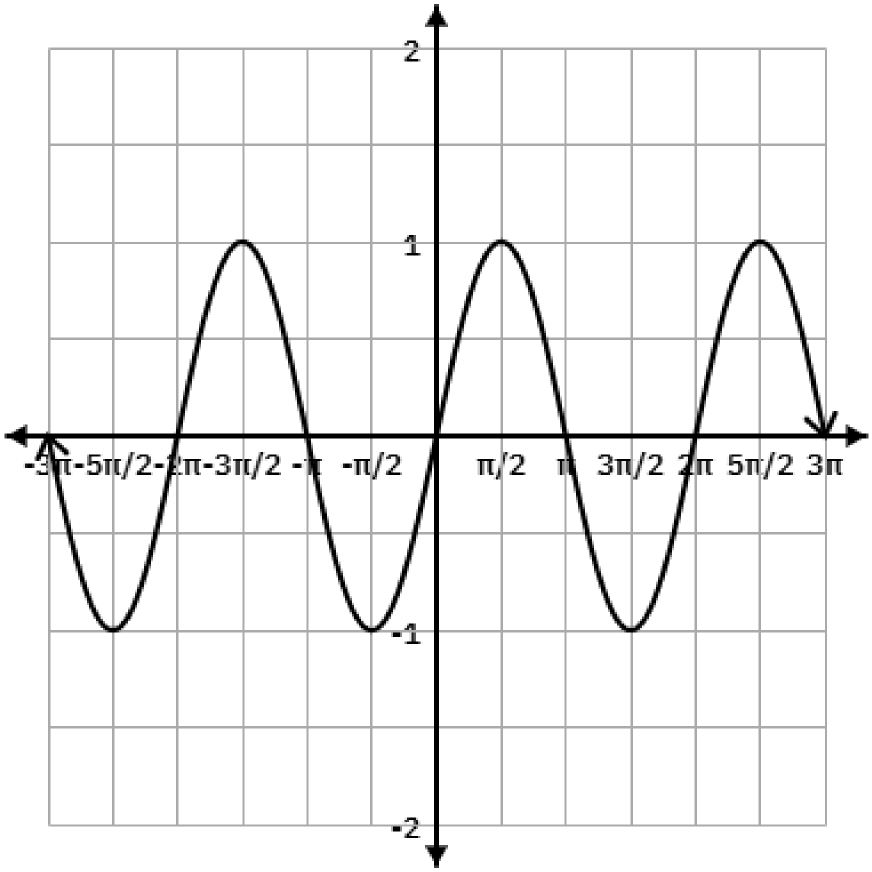


E) $(f \circ g)(-1)$

F) $(g \circ f)(-2)$

G) $(f \circ g^{-1})(1)$

H) $(g \circ f^{-1})(-1)$


2
1
-1
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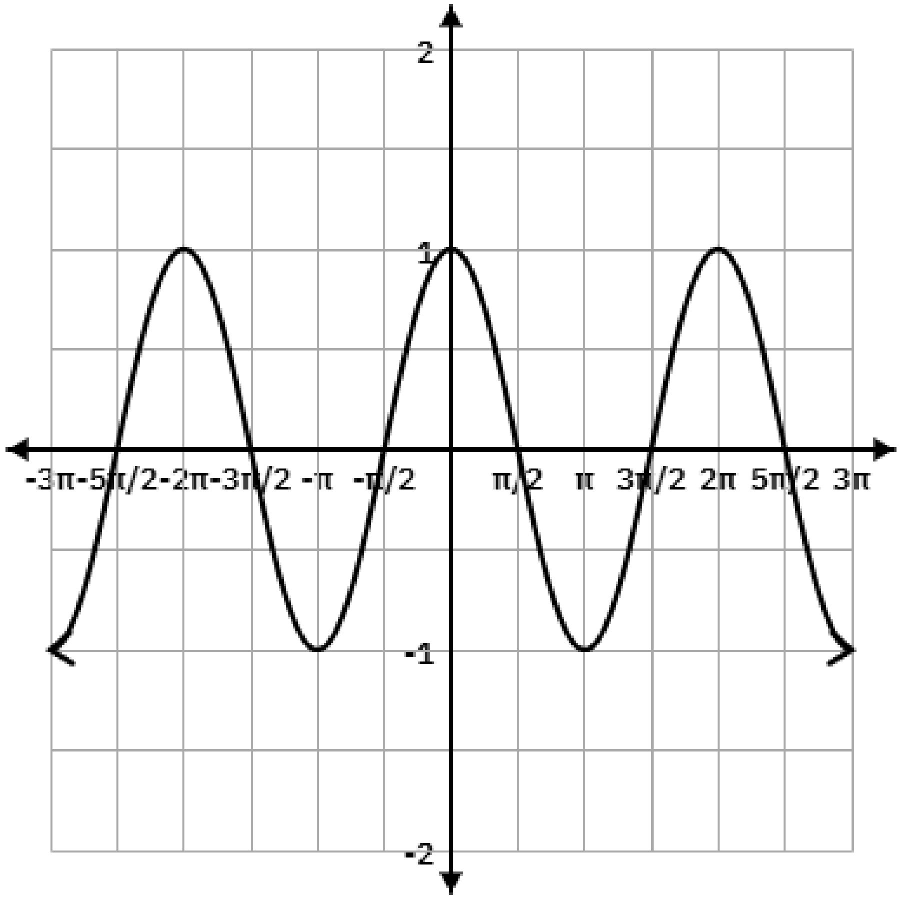
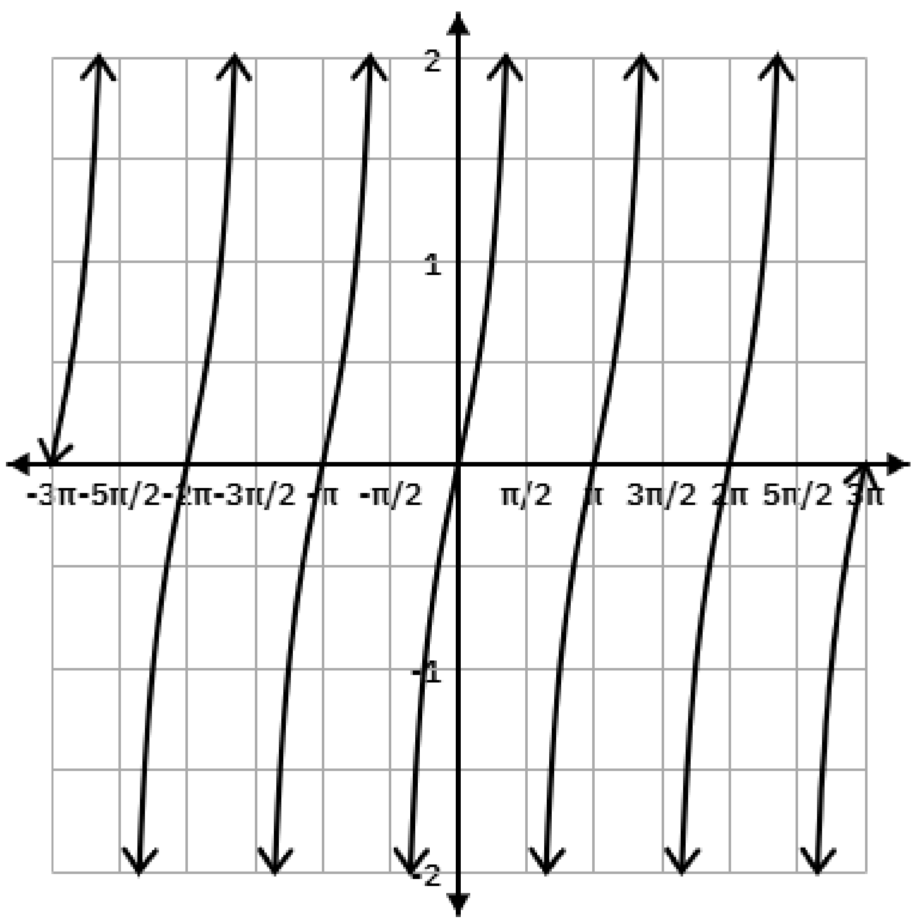
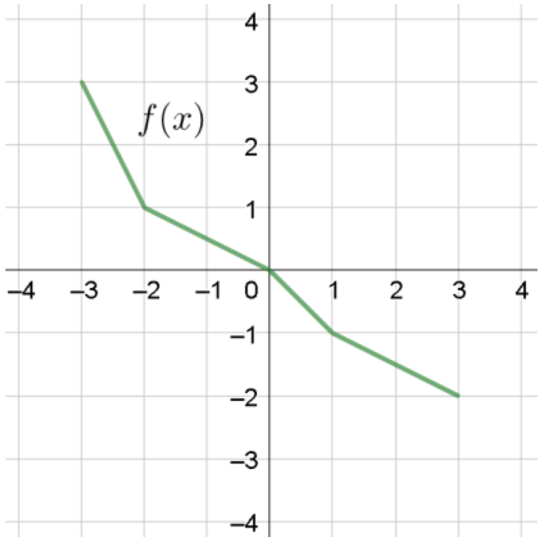


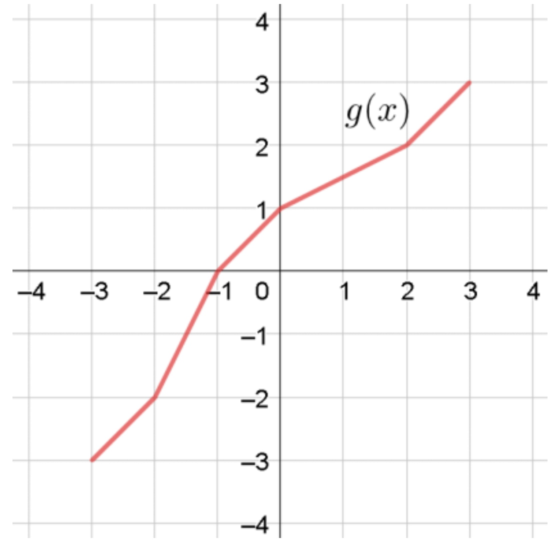
2
1
-1
-2
-3π -5π/2 -2π -3π/2 -π -π/2 π/2 π 3π/2 2π 5π/2 3π



2
1
-1
-2
-3π -5π/2 -2π -3π/2 -π -π/2 π/2 π 3π/2 2π 5π/2 3π

Find the indicated values if possible





I) $\left(f \circ f^{-1}\right)(2)$

J) $\left(g^{-1} \circ f^{-1}\right)(2)$

$f(x) = x + 4$ $g(x) = \sqrt{x} - 5$ $h(x) = x^2 + x, x \geq -\frac{1}{2}$

Find the indicated value.

A) $f^{-1}(0)$

B) $g^{-1}(2)$

C) $h^{-1}(2)$